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GAS-DYNAMIC PROCESSES IN TWO-PHASE FLOWS IN MHD GENERATORS

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A plane problem of a two-phase monodisperse flow of combustion products of plasma-forming composite solid propellants in the duct of a Faraday's MHD *generator with continuous electrodes, including an accelerating nozzle,* MHD *channel, and diffuser, is considered. An algorithm based on the pseudo-transient method is developed to solve the system of equations describing the two-phase flow. Gas-dynamic processes in the channels of the Pamir-1 setup are numerically studied. It is shown that shock-free deceleration of a supersonic flow to velocities close to the equilibrium velocity of sound in a two-phase mixture and significantly lower than the velocity of sound in the gas is possible in two-phase flows.*

Key words: MHD *generator, two-phase flow, numerical solution.*

Introduction. An analysis of experimental and theoretical results shows that gas-dynamic processes in pulsed MHD generators (PMHDG) on plasma-forming composite solid propellants have a spatial character. Even in linear PMHDGs, the flow can be non-one-dimensional because of the different values of the ponderomotive force and nonequilibrium distribution of particles over the channel cross section.

Non-one-dimensional plasma flows in MHD generators (MHDG) were mainly studied within the framework of the one-phase model [1–5]. There are some papers [6–8] dealing with certain aspects of non-one-dimensional two-phase MHD flows. In the case of strong MHD interaction, shock waves can be formed in the flow, which significantly affect the MHDG volt–ampere characteristics [4]. Such regimes of MHDG operation and associated features of two-dimensional two-phase magnetogasdynamic flows in MHD channels have not been adequately considered yet.

Formulation of the Problem and Initial System of Equations. We consider a plane flow of a mixture of low-temperature plasma (gas phase) and monodisperse particles in the gas-dynamic duct of an MHD setup including a Laval nozzle, a Faraday channel of the generator with continuous electrodes, and a diffuser. The two-phase flow is described with the help of the two-fluid model of continuous media [9], whose main assumptions for nozzle flows are formulated in [10]. In studying the two-phase flow in the MHDG, additional assumptions are made [6, 11].

The magnetic Reynolds number in the PMHDG is $\mathrm{Re_m} \ll 1$; therefore, the so-called galvanic approximation [6], in which the induced magnetic field is neglected, is used to calculate electromagnetic quantities. In addition, it is assumed that the induction vector of the external magnetic field has only one component: $\boldsymbol{B}(0, 0, B_Z(x))$.

Under the above-made assumptions, the gas-phase equations take the form

$$\frac{\partial}{\partial x}\boldsymbol{F} + \frac{\partial}{\partial y}\boldsymbol{G} = \boldsymbol{H},$$

$$\boldsymbol{F} = \begin{bmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ (e+P)u \end{bmatrix}, \qquad \boldsymbol{G} = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + P \\ (e+P)v \end{bmatrix},$$

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$$
\boldsymbol{H} = \begin{bmatrix} 0 \\ C_R\rho_{\mathrm{s}}(u_{\mathrm{s}}-u) + \dfrac{\sigma}{1+\beta^2}\Big(B_Z(\beta v - u) - \dfrac{V(t)}{h(x)}\Big)B_Z \\ C_R\rho_{\mathrm{s}}(v_{\mathrm{s}}-v) - \dfrac{\sigma}{1+\beta^2}\Big(B_Z(v + \beta u) + \beta\dfrac{V(t)}{h(x)}\Big)B_Z \\ C_\alpha\rho_{\mathrm{s}}(T_{\mathrm{s}}-T) + C_R\rho_{\mathrm{s}}[u_{\mathrm{s}}(u_{\mathrm{s}}-u) + v_{\mathrm{s}}(v_{\mathrm{s}}-v)] \\ - \dfrac{\sigma}{1+\beta^2}\Big(B_Z(\beta v - u) - \dfrac{V(t)}{h(x)}\Big)\dfrac{V(t)}{h(x)} \end{bmatrix}, \tag{1}
$$

$$
P = \rho R T, \qquad e = \rho(\varepsilon + (u^2+v^2)/2), \qquad \varepsilon = c_V T,
$$

$$
C_R = 18\eta(1 + 0.15\,\mathrm{Re}^{0.687})/(\rho_{\mathrm{sub}} d_{\mathrm{s}}^2), \qquad C_\alpha = 6\eta c_p(2 + 0{,}459\,\mathrm{Re}^{0.55}\,\mathrm{Pr}^{\,0.33})/(\rho_{\mathrm{sub}} d_{\mathrm{s}}^2 \mathrm{Pr}\,),
$$

and the condensed-phase equations have the form

$$
\frac{\partial}{\partial x}\,\rho_{\mathrm{s}} u_{\mathrm{s}} + \frac{\partial}{\partial y}\,\rho_{\mathrm{s}} v_{\mathrm{s}} = 0,
$$

$$
u_{\mathrm{s}}\frac{\partial}{\partial x}\,u_{\mathrm{s}} + v_{\mathrm{s}}\frac{\partial}{\partial y}\,u_{\mathrm{s}} = C_R(u - u_{\mathrm{s}}), \qquad u_{\mathrm{s}}\frac{\partial}{\partial x}\,v_{\mathrm{s}} + v_{\mathrm{s}}\frac{\partial}{\partial y}\,v_{\mathrm{s}} = C_R(v - v_{\mathrm{s}}), \tag{2}
$$

$$
u_{\mathrm{s}}\frac{\partial}{\partial x}\,T_{\mathrm{s}} + v_{\mathrm{s}}\frac{\partial}{\partial y}\,T_{\mathrm{s}} = \frac{C_\alpha}{c_{\mathrm{sub}}}(T - T_{\mathrm{s}}).
$$

Here ρ, P, T, $\boldsymbol{V}(u,v)$, e, and ε are the density, pressure, temperature, velocity vector, total energy of unit volume, and internal energy of unit mass of the gas phase, respectively (the corresponding parameters for solid particles are marked by the subscript "s"), R is the gas constant, $V(t)$ is the voltage on the electrodes, $h(x)$ is the distance between the electrodes, σ is the electrical conductivity, β is the Hall parameter, c_V and c_p are the specific heats of the gas at constant volume and pressure, respectively, C_R and C_α are the coefficients of force and temperature interaction between the gas phase and particles [10], η is the dynamic viscosity of the gas phase, Re and Pr are the Reynolds and Prandtl numbers, respectively, ρ_{sub} and c_{sub} are the specific density and heat capacity of the condensed-phase substance, respectively, and d_{s} is the diameter of particles of the condensed phase.

The elevated conductivity of the low-temperature plasma in the PMHDG is provided by introducing additives containing alkali metals with a reduced ionization potential into plasma-forming propellants. The following model dependences were obtained for such plasma in [2]:

$$
\sigma = \sigma_1\Big(\frac{P}{P_1}\Big)^{-0.5}\Big(\frac{T}{T_1}\Big)^{0.75}\exp\Big[\frac{I_e}{2}\Big(\frac{1}{T} - \frac{1}{T_1}\Big)\Big], \qquad \beta = \mu_{e1}\frac{P_1}{P}\Big(\frac{T}{T_1}\Big)^{0.5}|\boldsymbol{B}|. \tag{3}
$$

Here I_e is the ionization potential and μ_e is the mobility of electrons; the subscript 1 indicates some scale values of parameters.

For the external-load resistance R_{load}, the voltage on the electrodes is

$$
V(t) = \Big(-\int_0^L \frac{\sigma B_Z}{1+\beta^2}\,(u - \beta v)a(x)\,dx\Big)\Big/\Big(\frac{1}{R_{\mathrm{load}}} + \int_0^L \Big(\frac{\sigma}{1+\beta^2}\,\frac{a(x)}{h(x)}\Big)\,dx\Big),
$$

where $a(x)$ is the distance between the insulators and L is the electrode-zone length.

For the numerical solution of system (1), the steady subsystem of gas-phase equations is replaced by an unsteady system by adding the unsteady terms $\partial\boldsymbol{U}/\partial t$, where $\boldsymbol{U} = (\rho, \rho u, \rho v, e)^{\mathrm{t}}$. The flow rate, velocity-vector direction, and entropy were set for the unsteady subsystem of gas-phase equations in the input section of the gas-dynamic duct, and the no-slip conditions were set on the walls. The flow in the output section was assumed to be supersonic, and the boundary conditions were not imposed.

Owing to characteristic properties of system (2), the boundary conditions for the condensed phase were imposed only in the input section of the gas-dynamic duct: the velocity and temperature of particles were set from the condition of velocity and temperature equilibrium of the phases, and the density of the "gas" particles was assumed to be $\rho_{\mathrm{s}} = \rho Z/(1 - Z)$, where Z is the mass fraction of condensate.

It was assumed that liquid particles of metal oxides, hitting the channel contour, stick to the walls and, after cooling, form a thin film on the walls, which does not affect the flow in the main part of the channel. Owing to this assumption, the recoil and motion of particles along the walls were not considered.

The initial conditions were the values of the plasma parameters ρ, $\boldsymbol{V}(u, v)$, and e correlated with the boundary conditions and defined in a manner to obtain a supersonic flow in the MHD channel already at the initial time.

Pseudo-Transient Method for Calculating the Steady Two-Dimensional Two-Phase Magnetogasdynamic Flow. As the particle size $d_{\rm s}$ tends to zero, the interaction coefficients C_R and C_α tend to infinity, which is equivalent to emergence of small parameters at the derivatives in Eqs. (1) and (2). Therefore, the proposed algorithm of solving Eqs. (1) is based on a combination of the difference scheme implicit in the right sides [12] and the method developed by S. K. Godunov [13]. Godunov's method allows a correct calculation of shock waves, and the difference scheme implicit in the right sides allows one to solve equations with a small parameter at the derivative.

The conservation laws (1) in an integral form become

$$\oiint \boldsymbol{U}\,dx\,dy + \boldsymbol{F}\,dy\,dt + \boldsymbol{G}\,dt\,dx = \iiint\limits_V \boldsymbol{H}\,dx\,dy\,dt. \tag{4}$$

With accuracy to the terms τ, Δx, and Δy, the right side of Eq. (4) can be written as

$$\iiint\limits_V \boldsymbol{H}\,dx\,dy\,dt \approx \Omega_{jk}\tau\boldsymbol{H}^{n+1},$$

where Ω_{jk} is the cell area and τ is the step of integration in time.

The difference scheme for calculating parameters of the $(n+1)$th layer has the form

$$\rho^{n+1} = \rho^n + \Delta_\rho^n\tau/\Omega_{jk}; \tag{5}$$

$$\rho^{n+1}u^{n+1} = \rho^n u^n + \Delta_{\rho u}^n\tau/\Omega_{jk} + \Big\{B_Z\Big(\frac{\sigma}{1+\beta^2}\Big)^n\Big[B_Z(\beta^n v^{n+1} - u^{n+1}) - \frac{V(t)^n}{h(x)}\Big] + C_R^n\rho_{\rm s}(u_{\rm s} - u^{n+1})\Big\}\tau; \tag{6}$$

$$\rho^{n+1}v^{n+1} = \rho^n v^n + \Delta_{\rho v}^n\tau/\Omega_{jk} - \Big\{B_Z\Big(\frac{\sigma}{1+\beta^2}\Big)^n\Big[B_Z(v^{n+1} + \beta^n u^{n+1}) + \beta^n\frac{V(t)^n}{h(x)}\Big] + C_R^n\rho_{\rm s}(v_{\rm s} - v^{n+1})\Big\}\tau; \tag{7}$$

$$\begin{aligned}\rho^{n+1}\Big[c_V T^{n+1} + \Big(\frac{u^2+v^2}{2}\Big)^{n+1}\Big]v^{n+1} &= \rho^n\Big[c_V T^n + \Big(\frac{u^2+v^2}{2}\Big)^n\Big] + \frac{\Delta_H^n\tau}{\Omega_{jk}}\\ &- \Big\{\Big(\frac{\sigma}{1+\beta^2}\,\frac{V(t)}{h(x)}\Big)^n\Big[B_Z(\beta^n v^{n+1} - u^{n+1}) - \frac{V(t)^n}{h(x)}\Big]\Big\}\tau\\ &- \{C_\alpha^n\rho_{\rm s}(T_{\rm s} - T^{n+1}) + C_R^n\rho_{\rm s}[u_{\rm s}(u_{\rm s} - u^{n+1}) + v_{\rm s}(v_{\rm s} - v^{n+1})]\}\tau.\end{aligned} \tag{8}$$

Here Δ_ρ^n, $\Delta_{\rho u}^n$, $\Delta_{\rho v}^n$, and Δ_H^n are the total fluxes of mass, momentum projections, and enthalpy through the side surfaces of the cell, which are calculated by the scheme of [13].

The calculation sequence for the time t^{n+1} is as follows:

1) based on the discontinuity-decay scheme, the fluxes Δ_ρ^n, $\Delta_{\rho u}^n$, $\Delta_{\rho v}^n$, and Δ_H^n are calculated;
2) from Eq. (5), the density ρ^{n+1} is found;
3) by solving jointly Eqs. (6) and (7), u^{n+1} and v^{n+1} are found;
4) from Eq. (8), the temperature T^{n+1} is determined;
5) from the equation of state, the pressure P^{n+1} is found.

In solving the steady equations (2), we use the scheme of the method of characteristics, which is designed for solving equations with small parameters at the derivatives [9]. The plasma and particle parameters are jointly calculated by the pseudo-transient method in the following sequence: 1) stabilization of subsystem (1) is calculated; 2) Equations (2) are solved with allowance for the plasma parameters obtained. The particle parameters are interpolated to the nodes of the computational grid for the gas phase.

The process is repeated until the integral parameter $V(t)$ converges with given accuracy.

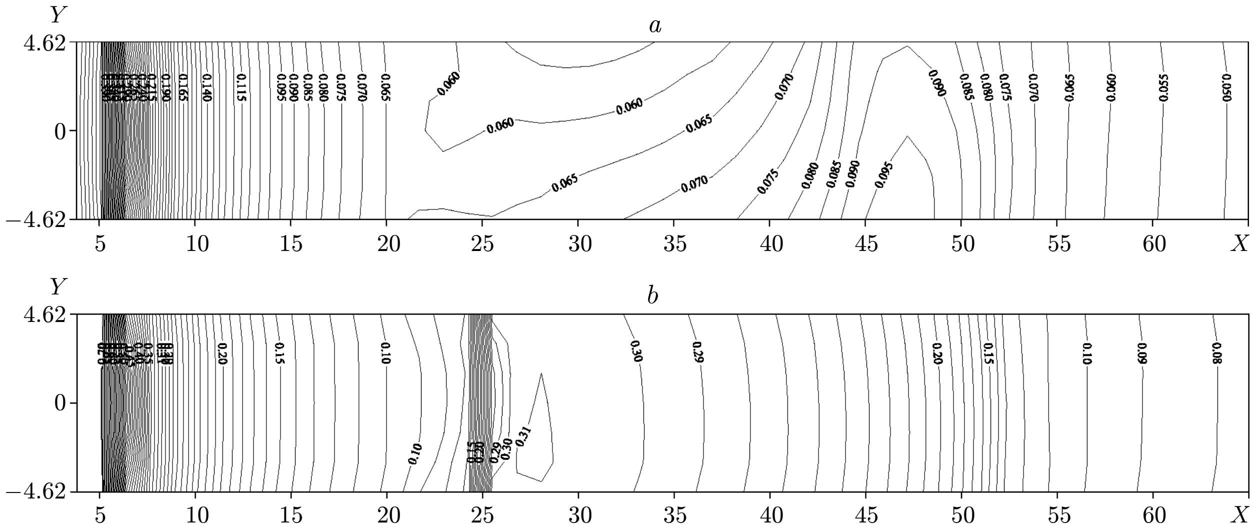

Fig. 1. Isobars for $Z = 0.238$, $B_Z = 3.5$ T, and $R_{\mathrm{load}} = 0.025$ (a) and 0.0083 Ω (b).

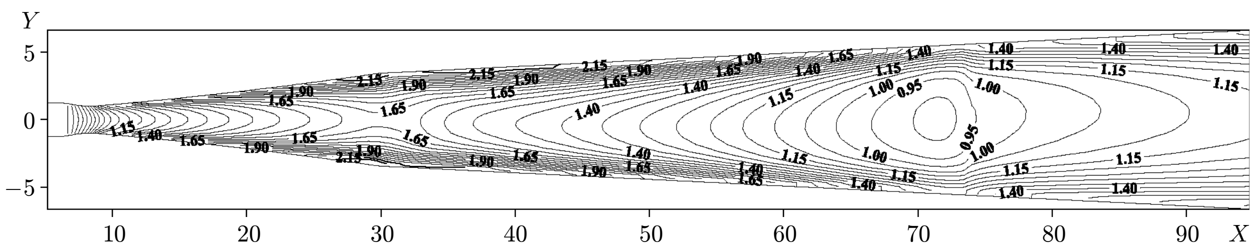

Fig. 2. Mach number isolines for $Z = 0.445$, $B_Z = 4$ T, and $R_{\mathrm{load}} = 0.025$ Ω.

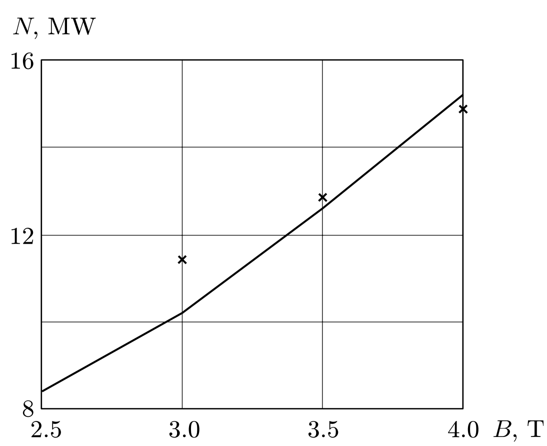

Fig. 3. Power versus magnetic induction for the Pamir-1 setup (the points show the experimental data of [2]).

Calculation Results for Two-Dimensional Two-Phase Magnetogasdynamic Flows. The calculations were performed in an MHD channel simulating the duct of the Pamir-1 setup. The influence of magnetic-field induction and external-load resistance on the flow parameters was considered. In calculating σ and β by formula (3), we assumed that $\sigma_1 = 55$ S/m, $\mu_{e1} = 0.17$ T^{-1}, $T_1 = 2774$ K, $P_1 = 3.86 \cdot 10^5$ N/m^2, and $I_e = 45\,300$ K [2]. Because of the large density of isolines of flow parameters in the nozzle, the region of the gas-dynamic duct of the setup was mapped onto a rectangle.

The isobars of the shock-free two-phase flow for $d_{\rm s} = 4 \cdot 10^{-6}$ m, $Z = 0.238$, $B_Z = 3.5$ T, $R_{\rm load} = 0.025\ \Omega$ are plotted in Fig 1a. As the load resistance decreases to $R_{\rm load} = 0.0083\ \Omega$, a shock wave arises in the flow (Fig. 1b).

The Mach number isolines in the channel of the Pamir-1 setup for $B_Z = 4$ T and $R_{\rm load} = 0.025\ \Omega$ are plotted in Fig. 2. For $B_Z > 3.7$ T, a shock-free subsonic flow arises in the channel. Nevertheless, the setup power for these values of induction continues to grow. Figure 3 shows the dependence of the MHDG power N on magnetic-field induction. The experimental data were taken from [2].

Conclusions. In the development of MHD generators, it is necessary to obtain a fully supersonic flow in the channel, since a shock wave responsible for boundary-layer separation can appear in the case of plasma deceleration to Mach numbers M < 1, which results in an increase in channel drag and a decrease in MHDG power. In addition, there are no mathematical tools for modeling transonic two-phase flows, hence, it is impossible to design generators with a mixed flow character in the channel.

In the case of a two-phase flow with high mass fractions of condensate in the MHD channel, it is possible to reach shock-free deceleration of a supersonic flow to M ≈ 0.8, which allows one to increase the power and efficiency of generators, which employ combustion products of metallized propellants as a working body.

It should also be noted that the increase in PMHDG power (for a PMHDG of the Pamir-1 type) is hindered by boundary-layer separation, which occurs when some critical value of the interaction parameter is exceeded. Therefore, it is impossible to increase the power of the setup by merely increasing magnetic-field induction. Still, the experiments of [2] showed that separation can be eliminated by boundary-layer suction.

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